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# A METHOD FOR DETERMINING ESTIMATES OF THE DYNAMIC UNBALANCE AND THE THRUST MISALIGNMENT OF SPIN-STABILIZED ROCKETS

W. J. Sacco

Department of the Army Project No. 503-06-002
Ordnance Management Structure Code No. 5010.11.812

BALLISTIC RESEARCH LABORATORIES

ABERDEEN PROVING GROUND, MARYLAND

B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

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A B E R D E E N   P R O V I N G   G R O U N D ,   M A R Y L A N D

B A L L I S T I C   R E S E A R C H   L A B O R A T O R I E S

MEMORANDUM REPORT NO. 1405

WJSacco/ic
Aberdeen Proving Ground, Md.
May 1962

# A METHOD FOR DETERMINING ESTIMATES OF THE DYNAMIC UNBALANCE AND THE THRUST MISALIGNMENT OF SPIN-STABILIZED ROCKETS

## ABSTRACT

Solutions to the equation of motion of a spinner artillery rocket about its center of gravity under the influence of thrust misalignment and dynamic unbalance are obtained. A method for determining effective values of dynamic unbalance and thrust misalignment is given.

## INTRODUCTION

In the mathematical theory of the motion of spin-stabilized rockets during the burning phase, the mathematical model is a system of ordinary linear differential equations with variable coefficients and various forcing terms. Solution of the differential equations involves contributions due to unit values of both the initial parameters and coefficients of the forcing functions. We shall be concerned here only with the angular motion about the center of gravity and with forcing terms involving only the thrust misalignment and the dynamic unbalance.

The motion of a spinner rocket is similar to that of a spinning top or gyroscope. In addition to the spinning motion there exists a precession and nutation about the center of gravity. The thrust misalignment, $L_c$ and the dynamic unbalance, $\beta_c$, are the principal contributors to the transverse angular motion of the rocket. In fact, in the second part of this report we shall postulate, for an idealized situation, that $L_c$ and $\beta_c$ are the sole contributors to the transverse angular motion; thereby, enabling us to give a method for determining the approximate average values of $L_c$ and $\beta_c$.

The coordinate system which we shall use to describe the motion of the spinning rocket is shown in Figure I. OXYZ is a right-handed system of axes with the origin O at the center of gravity of the projectile. The reference axis OX is taken in the initial direction of the rocket axis, OY upward in the vertical plane, through OX and OZ to the right as viewed from the rear of the projectile.

We can imagine a sphere with the center at the center of gravity and whose radius is equal to the distance from the center of gravity to the nose of the projectile, and then picture a plane perpendicular to OX and tangent to the sphere. The nose of the projectile can be thought of as tracing out a curve in this plane, and we let the quantities $\phi_R$ and $\phi_I$ be the rectangular coordinates of a point on the curve.

We can treat the plane as a complex plane, with the real axis parallel to OY, and the imaginary axis parallel to OZ, and the angles measured clockwise from the vertical as viewed from the rear of the rocket. In this

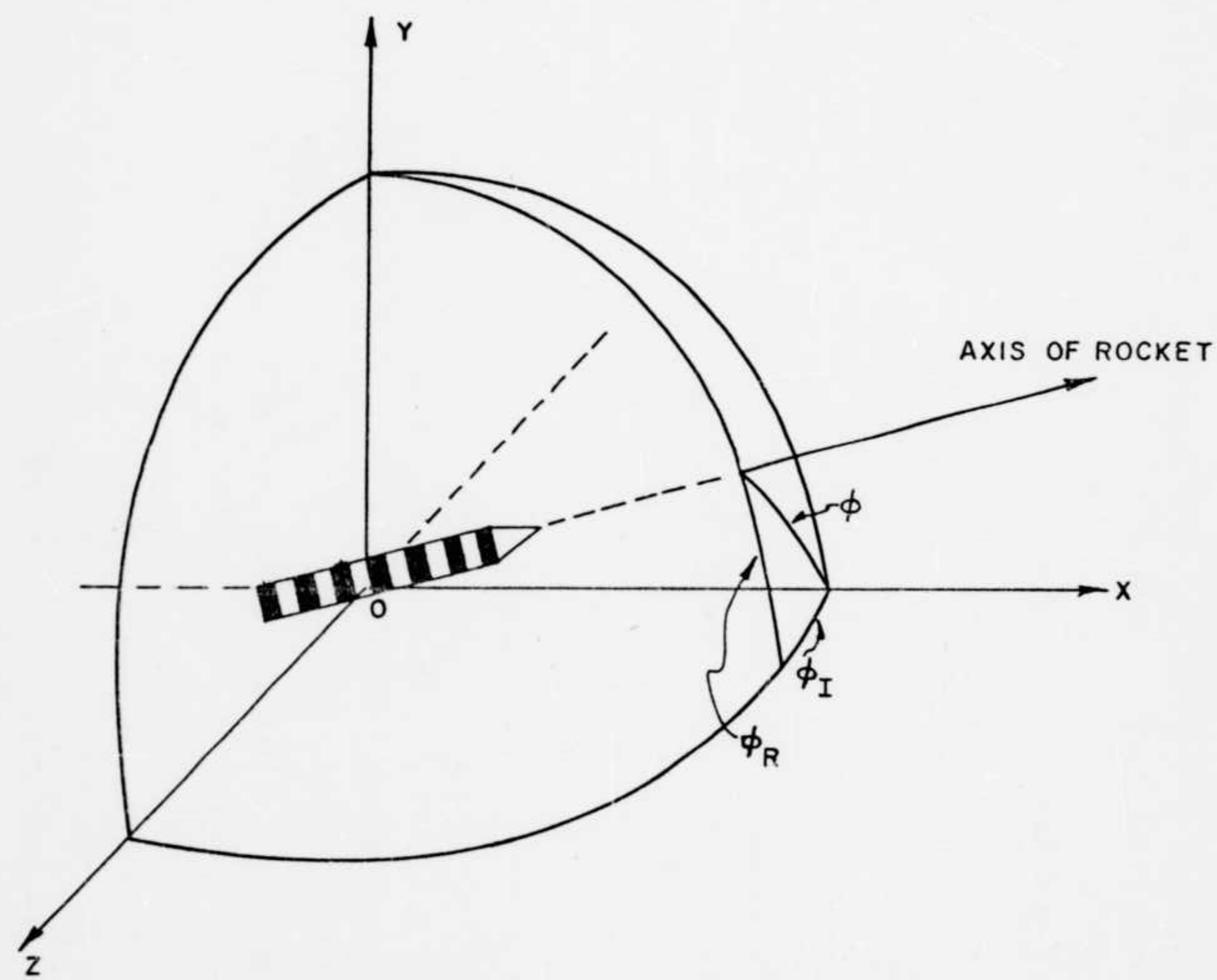


FIGURE I-COORDINATE SYSTEM

representation the complex number $\phi$ defined by $\phi = \phi_R + i\phi_I$ specifies the orientation of the instantaneous axis with respect to the initial axis of the rocket. It is clear that $|\phi|$ is the sine of the angle between the two directions. We shall assume that $|\phi|$ is less than 0.1 radians for all time values during burning, which enables us to use the approximations of replacing the sine of the angle by the angle expressed in radians and the cosine by 1. Experimentation has shown that these approximations are valid.

During the burning period of a spinning rocket the gas flow through the canted nozzles produces the forward thrust on the rocket and the torque about the longitudinal axis which produces the axial spin.

Generally the line of thrust of the rocket will not pass through the center of gravity of the rocket but will miss it by a distance $L_c$. This produces a torque about the center of gravity resulting in a cross spin. The distance $L_c$ is the so-called the thrust misalignment.

The axis corresponding to the least polar moment of inertia is called the principal longitudinal axis of inertia. Ideally it should coincide with the geometrical axis of symmetry of the rocket. When the principal longitudinal axis of inertia of the spinning rocket makes a complex angle, $\beta_c$, with the geometrical axis, the resulting effect is characterized by a moment about an axis perpendicular to the rocket axis. The angle $\beta_c$ is called the dynamic unbalance angle.

For spin-stabilized rockets the major lateral dispersion-producing factors are the (1) dynamic unbalance, (2) linear thrust misalignment, (3) initial yaw, (4) the initial transverse angular velocity, and (5) variable winds. For the idealized case considered here, the last three factors are eliminated.

The equation of motion considered here describes the angular rotation of the rocket about an instantaneous transverse axis through the center of gravity, when the center of gravity is fixed in space. The equation which is derived in references [H] and [CH], has the form

$$\ddot{\phi} - 2iq\alpha t\dot{\phi} = f(t) \qquad (1)$$

where

$i = \sqrt{-1}$ ,

$\phi$ = complex orientation angle ,

$2q$ = ratio of axial and transverse moments of inertia,

$\alpha$ = angular acceleration of the rocket about its longitudinal axis,

$t$ = time measured from ignition,

$f(t)$ = resultant of cross-torques due to linear thrust misalignment and dynamic unbalance, i.e.

$$f(t) = \begin{cases} \dfrac{-TL_c e^{i\alpha t^2}}{} & \text{(due to thrust misalignment)} \\ -(1-2q)\beta_c \left[ i\alpha-(\alpha t)^2 \right] e^{i\alpha t^2} & \text{(dynamic unbalance),} \end{cases}$$

$B$ = transverse moment of inertia,

and

$T$ = thrust of rocket .

In these representations, $L_c$ and $\beta_c$ are complex parameters which characterize the linear thrust misalignment and the dynamic unbalance respectively.

Equation (1) is a linear differential equation with variable coefficients. For our purposes here, $q, \alpha, B, T, |\beta_c|$, and $|L_c|$ are assumed to be constants.

## SOLUTIONS TO THE EQUATION OF MOTION

We perform two integrations on equation (1), assuming initially that $\Phi(t_o) = \dot{\Phi}(t_o) = 0$, and obtain the expression

$$\Phi(t) = \int_{t_o}^{t} e^{iq\alpha x^2} \int_{t_o}^{x} f(u) e^{-iq\alpha u^2} \, du dx \tag{2}$$

Case a) The thrust misalignment.

We let $\Phi_1(t)$ equal the $\Phi(t)$ of Equation (2) when

$$f(u) = -\frac{TL_c}{B} e^{i\alpha u^2}$$

where $f(u)$ is the forcing function for the thrust misalignment.

We get

$$\Phi_1(t) = \int_{t_o}^{t} e^{iq\alpha x^2} \int_{t_o}^{x} -\left(\frac{TL_c}{B}\right) e^{i\alpha u^2} e^{-i\alpha q u^2} \, dudx \quad ,$$

$$= -\frac{TL_c}{B} \int_{t_o}^{t} e^{iq\alpha x^2} \int_{t_o}^{x} e^{i\alpha(1-q)u^2} \, dudx \quad . \tag{3}$$

Letting $A = \alpha q$ and $D = \alpha(1-q)$, then Equation (3) reduces to

$$\Phi_1(t) = -\frac{TL_c}{B} \int_{t_o}^{t} e^{iAx^2} \int_{t_o}^{x} e^{iDu^2} dudx \quad . \tag{4}$$

Case b) Dynamic Unbalance

We let $\Phi_2(t)$ equal the $\Phi(t)$ of Equation (2) when

$$f(u) = -(1-2q)\beta_c e^{i\alpha t^2} (i\alpha - (\alpha t)^2) \quad ,$$

where $f(u)$ is the forcing function for dynamic unbalance.

We obtain

$$\Phi_2(t) = \int_{t_o}^{t} e^{iq\alpha x^2} \int_{t_o}^{x} -(1-2q)\beta_c e^{i\alpha u^2} (i\alpha - (\alpha u)^2) e^{-i\alpha q u^2} du \, dx.$$

Again letting $A = \alpha q$, $D = \alpha(1-q)$, and rearranging terms we get

$$\Phi_2(t) = -(1-2q)\beta_c \left( i\alpha \int_{t_o}^{t} e^{iAx^2} \int_{t_o}^{x} e^{iDu^2} \, du \, dx \right.$$

$$\left. -\alpha^2 \int_{t_o}^{t} e^{iAx^2} \int_{t_o}^{x} u^2 e^{iDu^2} \, du \, dx \right) . \tag{5}$$

A look at Equation (4) and Equation (5) indicates that we are faced with the evaluation of the following integrals:

$$F_1(t) \equiv \int_{t_o}^{t} e^{iAx^2} \int_{t_o}^{x} e^{iDu^2} \, du \, dx , \tag{6}$$

and

$$F_2(t) \equiv \int_{t_o}^{t} e^{iAx^2} \int_{t_o}^{x} u^2 e^{iDu^2} \, du \, dx \tag{7}$$

Then we have

$$\phi_1(t) = -\frac{TL_c}{B} F_1(t) , \tag{8}$$

and

$$\phi_2(t) = -(1-2q)\beta_c (i\alpha F_1(t) - \alpha^2 F_2(t)) . \tag{9}$$

We proceed to evaluate $F_1(t)$ and $F_2(t)$. Define

$$F_3(x) \equiv \int_{t_o}^{x} e^{iDu^2} \, du . \tag{10}$$

Letting $x = ms$ where $m = \sqrt{\frac{\pi}{2D}}$ we obtain

$$F_3\left(\frac{x}{m}\right) = m \int_{t_o/m}^{x/m} e^{i\pi s^2/2} \, ds,$$

$$= m \left( \int_{t_o/m}^{x/m} \cos \frac{\pi}{2} s^2 \, ds + i \int_{t_o/m}^{x/m} \sin \frac{\pi}{2} s^2 \, ds \right) \tag{11}$$

$$= m \left[ C(x/m) - C(t_o/m) + i \left( S(x/m) - S(x_o/m) \right) \right]$$

where

$$C(r) = \int_0^r \cos \frac{\pi}{2} s^2 \, ds \quad ,$$

and

$$S(r) = \int_0^r \sin \frac{\pi}{2} s^2 \, ds \quad .$$

$C(r)$ and $S(r)$ are the well - tabulated Fresnel Integrals. [WS]

Using Equation (11) and Equation (6) we obtain

$$F_1(t) = m \int_{t_o}^{t} e^{iAx^2} \left[ C(x/m) - C(t_o/m) + i(S(x/m) - S(x_o/m)) \right] dx \tag{12}$$

Using the identity,

$$e^{iAx^2} = \cos Ax^2 + i \sin Ax^2 \quad ,$$

and separating the right side of Equation (12) into a sum of integrals, we obtain

$$\begin{aligned} F_1(t) = m \Bigg[ & \int_{t_o}^{t} C(x/m) \cos Ax^2 \, dx - C(t_o/m) \int_{t_o}^{t} \cos Ax^2 \, dx \\ & + i \int_{t_o}^{t} S(x/m) \cos Ax^2 dx - iS(t_o/m) \int_{t_o}^{t} \cos Ax^2 \, dx \\ & + i \int_{t_o}^{t} C(x/m) \sin Ax^2 dx - iC(t_o/m) \int_{t_o}^{t} \sin Ax^2 \, dx \\ & - \int_{t_o}^{t} S(x/m) \sin Ax^2 dx + S(t_o/m) \int_{t_o}^{t} \sin Ax^2 \, dx \Bigg] \end{aligned} \tag{13}$$

We can rewrite $F_1(t)$ as

$$F_1(t) = m(I_1 - I_2 + i(I_3 + I_4)) \tag{14}$$

where

$$I_1 = \int_{t_o}^{t} \cos Ax^2(C(x/m) - C(t_o/m))dx \quad , \tag{15}$$

$$I_2 = \int_{t_o}^{t} \sin Ax^2(S(x/m) - S(t_o/m))dx \quad , \tag{16}$$

$$I_3 = \int_{t_o}^{t} \cos Ax^2(S(x/m) - S(t_o/m))dx \quad , \tag{17}$$

and

$$I_4 = \int_{t_o}^{t} \sin Ax^2(C(x/m) - C(t_o/m))dx \quad . \tag{18}$$

To evaluate $F_2(t)$ as defined in (7) introduce the notation

$$F_4(x) \equiv \int_{t_o}^{x} u^2 e^{iDu^2} du \quad . \tag{19}$$

Letting $r = u$, $dv = ue^{iDu^2} du$, and integrating by parts, we obtain

$$F_4(x) = - \frac{i}{2D} (xe^{iDx^2} - t_o e^{iDt_o^2}) + \frac{i}{2D} \int_{t_o}^{x} e^{iDu^2} du \quad ,$$

$$= \frac{i}{2D} \Big[ (t_o e^{iDt_o^2} - xe^{iDx^2}) + m(C(x/m) - C(t_o/m) \tag{20}$$

$$+ \quad iS(x/m) - iS(t_o/m)) \Big]$$

We can use $F_4(x)$ in evaluating $F_2(t)$.

$$F_2(t) = \int_{t_o}^{t} e^{iAx^2} F_4(x)dx \tag{21}$$

$$= \frac{i}{2D}\left[ t_o e^{iDt_o^2} \int_{t_o}^{t} e^{iAx^2} dx - \int_{t_o}^{t} x e^{i(A+D)x^2} dx \right.$$

$$\left. + m \int_{t_o}^{t} e^{iAx^2} (C(x/m) - C(t_o/m) + iS(x/m) - iS(t_o/m)) dx \right]$$

We now evaluate the three terms present on the right side of Equation (21). Using the identities

$$e^{iAx^2} = \cos Ax^2 + i \sin Ax^2 \quad ,$$

and

$$e^{iDt_o^2} = \cos Dt_o^2 + i \sin Dt_o^2 \ ,$$

we get

$$t_o e^{iDt_o^2} \int_{t_o}^{t} e^{iAx^2} dx = t_o(\cos Dt_o^2 + i \sin Dt_o^2)$$

$$\left( \int_{t_o}^{t} \cos Ax^2 dx + i \int_{t_o}^{t} \sin Ax^2 dx \right) \qquad (22)$$

$$= t_o \cos Dt_o^2 \int_{t_o}^{t} \cos Ax^2 dx - t_o \sin Dt_o^2 \int_{t_o}^{t} \sin Ax^2 dx)$$

$$+ i(t_o \cos Dt_o^2 \int_{t_o}^{t} \sin Ax^2 dx + t_o \sin Dt_o^2 \int_{t_o}^{t} \cos Ax^2 dx).$$

The second integral on the right side of Equation (21) is

$$\int_{t_o}^{t} x e^{i(A+D)x^2} dx \ .$$

Multiplying and dividing by $2i(A+D)$ we obtain

$$\frac{1}{2i(A+D)} \int_{t_o}^{t} e^{i(A+D)x^2} (2i(A+D)x) dx$$

$$= \frac{1}{2i(A+D)} \Big|_{t_o}^{t} e^{i(A+D)x^2} \tag{23}$$

$$= \frac{1}{2i(A+D)} \left(\cos(A+D)t^2 - \cos(A+D)t_o^{\,2}\right)$$

$$+ \frac{1}{2(A+D)} \left(\sin(A+D)t^2 - \sin(A+D)t_o^{\,2}\right) .$$

The third integral on the right side of Equation (21) is equal to

$$m \int_{t_o}^{t} (\cos Ax^2 + i \sin Ax^2) \Big[ C(x/m) - C(t_o/m) + i(S(x/m) \tag{24}$$

$$- S(t_o/m))\Big] dx = m(I_1 - I_2 + i(I_3 + I_4)) .$$

Let

$$I_5 = \int_{t_o}^{t} \cos Ax^2 dx \quad , \tag{25}$$

and

$$I_6 = \int_{t_o}^{t} \sin Ax^2 dx \quad . \tag{26}$$

Combining information from Equations (21), (22), (23), (24), (25) and (26), we find

$$F_2(t) = \frac{i}{2D} \Big[ t_o \cos Dt_o^{\,2} I_5 + it_o \cos Dt_o^{\,2} I_6 + it_o \sin Dt_o^{\,2} I_5$$

$$- t_o \sin Dt_o^{\,2} I_6 - \frac{i}{2i(A+D)} (\cos(A+D)t^2 - \cos(A+D)t_o^{\,2})$$

$$- \frac{1}{2(A+D)} (\sin(A+D)t^2 - \sin(A+D)t_o^{\,2}) + m(I_1 - I_2 + i(I_3 + I_4)) \Big] ,$$

$$= - \frac{t_o}{2D} \cos Dt_o^{\,2} I_6 - \frac{t_o}{2D} \sin Dt_o^{\,2} I_5 - \frac{1}{4D(A+D)}$$

$$(\cos(A+D)t^2 - \cos(A+D)t_o^{\,2}) \tag{27}$$

$$- \frac{m}{2D} (I_3 + I_4) + i \left[ \frac{t_o}{2D} \cos Dt_o^2 I_5 - \frac{t_o}{2D} \sin Dt_o^2 I_6 \right.$$

$$\left. - \frac{1}{4D(A+D)} (\sin(A+D)t^2 - \sin(A+D)t_o^2) + \frac{m}{2D}(I_1 - I_2) \right].$$

Using Equation (14) in Equation (8) we obtain

$$\frac{\Phi_1(t)}{L_c} = - \frac{Tm}{B} (I_1 - I_2 + i(I_3 + I_4)). \tag{28}$$

We may write with the aid of Equations (9), (14), and (27)

$$\begin{aligned} \frac{\Phi_2(t)}{\beta_c} = {} & (1-2q)\, \alpha \sqrt{\frac{\pi}{2D}} (I_3 + I_4) - \frac{\alpha^2(1-2q)}{2D} t_o(\cos Dt_o^2 I_6 \\ & + \sin Dt_o^2 I_5) - \frac{(1-2q)\alpha^2}{4D(A+D)} (\cos(A+D)t^2 - \cos(A+D)t_o^2) \\ & - \frac{(1-2q)\alpha^2}{2D} \sqrt{\frac{\pi}{2D}} (I_3 + I_4) \\ & + i \left[ -(1-2q)\alpha \sqrt{\frac{\pi}{2D}} (I_1 - I_2) + \frac{(1-2q)\alpha^2 t_o}{2D} (\cos Dt_o^2 I_5 \right. \\ & - \sin Dt_o^2 I_6) - \frac{(1-2q)\alpha^2}{4D(A+D)} (\sin(A+D)t^2 - \sin(A+D)t_o^2) \\ & \left. + \frac{(1-2q)\alpha^2}{2D} \sqrt{\frac{\pi}{2D}} (I_1 - I_2) \right]. \end{aligned} \tag{29}$$

## A METHOD FOR DETERMINING EFFECTIVE VALUES OF DYNAMIC UNBALANCE AND THRUST MISALIGNMENT

We could idealize the physical setup by imagining the projectile with its center of gravity somehow held fixed in space, but with the projectile free to move about its center of gravity. If we assume that the transverse angular motion is caused entirely by thrust misalignment and dynamic unbalance, solutions to Equation (2) can be used to provide an indirect method of estimating $L_c$ and $\beta_c$. Indeed, for positive unit values of $L_c$ and $\beta_c$ we could obtain the solutions

$\frac{\phi_1(t)}{L_c}$ and $\frac{\phi_2(t)}{\beta_c}$ for discrete values of t between the ignition and the end of burning times. The solutions of Equation (2) would be determined either by solving it directly by numerical methods or from Equations (28) and (29).

We would then obtain an over-determined system of equations.

$$
\begin{aligned}
\phi_R(t_1) &= x(t_1)\mathrm{Re}(L_c) + y(t_1)\mathrm{Re}(\beta_c) \quad , \\
\phi_R(t_2) &= x(t_2)\mathrm{Re}(L_c) + y(t_2)\mathrm{Re}(\beta_c) \quad , \\
&\vdots \\
\phi_R(t_n) &= x(t_n)\mathrm{Re}(L_c) + y(t_n)\mathrm{Re}(\beta_c) \quad ,
\end{aligned}
\tag{30}
$$

where

$$x(t) = \text{real component of } \frac{\phi_1(t)}{L_c} \quad ,$$

$$y(t) = \text{real component of } \frac{\phi_2(t)}{\beta_c} \quad ,$$

$Re(L_c)$ = real component of $L_c$ ,

$Re(\beta_c)$ = real component of $\beta_c$ .

As mentioned above, $x(t_i)$ and $y(t_i)$ are obtained from the solutions of Equation (2). The angles $\phi(t_i)$ would be measured experimentally (perhaps by a transducer). The values of $Re(L_c)$ and $Re(\beta_c)$ would be unknown. Since we could expect errors in the $\phi$, x, and y values, estimates of $Re(L_c)$ and $Re(\beta_c)$ would be obtained by applying least squares techniques. Similar expressions would provide estimates for the imaginary components of $L_c$ and $\beta_c$.

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# BIBLIOGRAPHY

[CH] Cell, J. and Harrington, W. The Motion of a Spin-Stabilized Rocket with constant angular and linear accelerations during burning outside the Launcher. North Carolina State College: ERD-82/8, 1958.

[ H] Harrington, W. Mathematical Studies of the Motion of a Spin-Stabilized Rocket During the Burning Period. North Carolina College: ERD-64/2, 1955.

[WS] Wijngaarden, A. and Scheen, W., 1949. Table of Fresnel Integrals. Report R49. Computation Department of the Mathematical Center at Amsterdam.

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Bodies of revolution - Motion

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